

Pre Calculus 11: Chapter 3- Solving Quadratic Equations Review

1. Factor the following expressions.

a) $x^2 + 2x - 15$

$$= (x+5)(x-3)$$

b) $6x^2 + 19x - 7$

$$= (3x-1)(2x+7)$$

c) $8x^2 - 18$

$$= 2(2x+3)(2x-3)$$

d) $2(2x+1)^2 - 9(2x+1) - 5$

$$= 2(4x+3)(x-2)$$

e) $4x^2 - \frac{1}{16}y^2$

$$= (2x)^2 - \left(\frac{1}{4}y\right)^2$$

$$= (2x + \frac{1}{4}y)(2x - \frac{1}{4}y)$$

2. Solve the following equations.

a) $x - 5 = \sqrt{20 - 2x}$

$$(x-5)^2 = 20 - 2x$$

$$x^2 - 10x + 25 + 2x - 20 = 0$$

$$x^2 - 8x + 5 = 0$$

$$x = \frac{8 \pm \sqrt{64 - 20}}{2}$$

$$x = \frac{8 \pm \sqrt{44}}{2} = 4 \pm \sqrt{11}$$

c) $(x-3)^2 - 5 = 12$

$$x-3 = \pm\sqrt{17}$$

$$x = 3 \pm \sqrt{17}$$

b) $4x(5x-15) = 0$

$$20x^2 - 60x = 0$$

$$20x(x-3) = 0$$

$$\begin{cases} 20x = 0 \\ x-3 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 3 \end{cases}$$

OR simply: (Already factored)

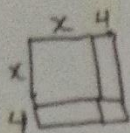
$$\begin{cases} 4x = 0 \\ 5x-15 = 0 \end{cases} \Rightarrow \begin{cases} x_1 = 0 \\ x_2 = 3 \end{cases}$$

3. Determine the value of p that makes $x^2 + 6x + p$ a perfect square.

$$\boxed{p=9}$$

Why? $x^2 + 6x + 3^2$
 $x^2 + 6x + 9 =$

4. A square plot of land is to be expanded. The length of each side is to be increased by 4 m. The area of the new land will be 225 m^2 . Determine the **perimeter** of the new plot of land.



$$(x+4)(x+4) = 225$$

$$x^2 + 8x + 16 = 225$$

$$x^2 + 8x - 209 = 0$$

$$x = \frac{-8 \pm \sqrt{64 - 4(-209)}}{2}$$

$$x = \frac{-8 \pm 30}{2} \Rightarrow \begin{cases} x_1 = -19 \\ x_2 = 11 \end{cases} \leftarrow \text{choose positive}$$

New side length of land is 11 m.

$$P = 11 + 11 + 11 + 11 = 44 \text{ m}$$

∴ The perimeter of the new plot of land is 44 m

5. Solve for y . $y^2 + 4y = x - 10$ (HINT: Complete the square!)

$$y^2 + 4y + 2^2 = x - 10 + 4$$

$$(y+2)^2 = x-6$$

$$y+2 = \pm \sqrt{x-6}$$

$$y = -2 \pm \sqrt{x-6}$$

$$\therefore y = -2 + \sqrt{x-6} \text{ or } y = -2 - \sqrt{x-6}$$

6. Solve $3x^2 - 3x - 5 = 0$

$$x = \frac{3 \pm \sqrt{9 - 4(3)(-5)}}{6}$$

$$\boxed{x = \frac{3 \pm \sqrt{69}}{6}}$$

7. $x^2 + 3x + d = 0$ has only one root. Use the quadratic formula to determine the value of d .

$$b^2 - 4ac = 0$$

$$(3)^2 - 4(1)(d) = 0$$

$$\frac{9}{4} = \frac{4d}{4}$$

$$d = \frac{9}{4}$$

$$\therefore d = \frac{9}{4}$$

8. What does it mean if $b^2 - 4ac > 0$?

If $b^2 - 4ac > 0$ we have a positive discriminant. This means the equation has two real roots!

9. The difference between the squares of two numbers is 75. One number is double the other. What is the product of the two numbers?

Let x be one number and $2x$ be the other number.

$$(2x)^2 - x^2 = 75$$

$$4x^2 - x^2 = 75$$

$$\frac{3x^2}{3} = \frac{75}{3}$$

$$x^2 = 25$$

$$x = \pm 5$$

$$\therefore 2x = 2(\pm 5) = \pm 10$$

$$x = 5$$
$$2x = 10$$

or

$$x = -5$$
$$2x = -10$$

$$\text{Product: } 5 \times 10 = 50$$
$$-5 \times -10 = 50$$

\therefore The product of the numbers is 50

10. Factor $x^3 - 3x^2 + 5x - 15$.

$$= (x^2 + 5)(x - 3)$$

Challenge! 😊

How?

x^2	5
x	-3

$$-3x^2 + 5x \quad \checkmark$$